observe:

$$1 \to \Pi_{2/1} \to \Pi_2 \xrightarrow{p} \Pi_1 \to 1$$

center-free

$g \in \Pi_2$

Let $\alpha \in \mathrm{Aut}^{FC}(\Pi_1)$

$\beta, \gamma \in \mathrm{Aut}^{FC}(\Pi_2)$ s.t. ① $\gamma^{-1} \circ \beta = \mathrm{Inn}(g)$

② $\beta_1 = \alpha, \ \gamma_1 = \alpha$

$\Rightarrow (\gamma^{-1} \circ \beta)_1 = id$

$\Rightarrow p(g) \in Z(\Pi_1) = \{1\}$

$\Rightarrow \gamma|_{\Pi_{2/1}}, \ \beta|_{\Pi_{2/1}}$ determine same $\in \mathrm{Out}^{FC}(\Pi_{2/1})$

$g \in \Pi_2$

$= \mathrm{Inn}(g)$

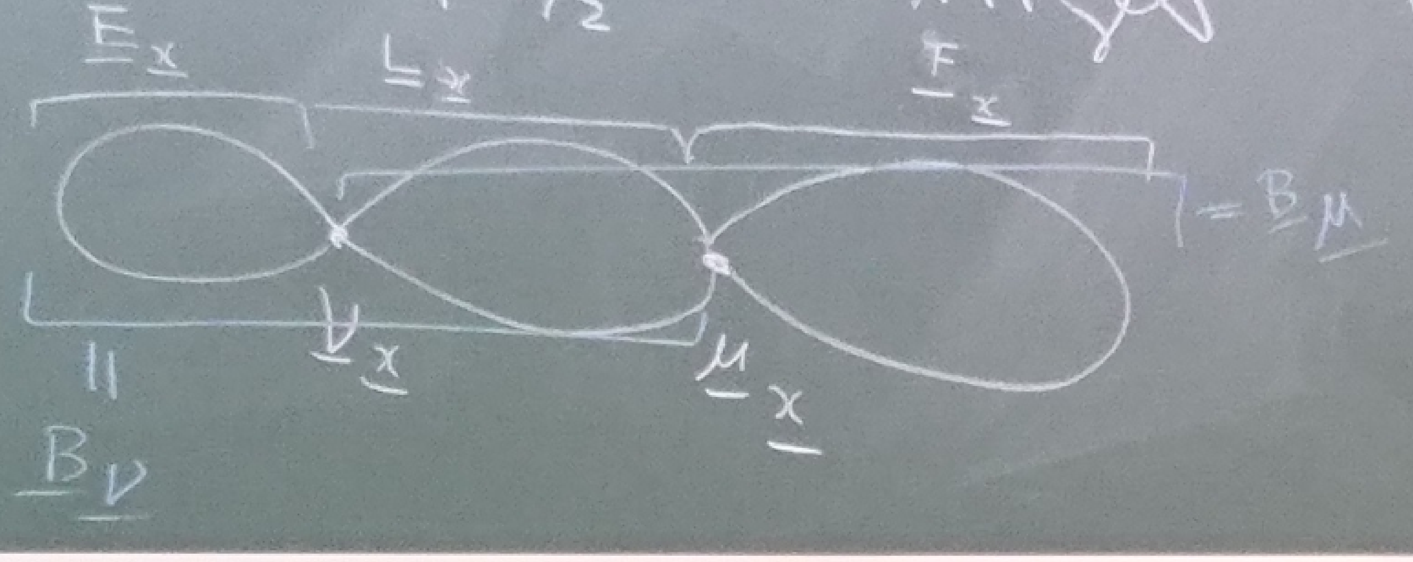
$\gamma_1 = \alpha$

$\frac{3}{1})$

$$\beta_{3/2}^{\underline{\nu}} := \beta_2^{\mathrm{tpd}}\Big|_{\Pi_{3/1}^{\mathrm{tpd}}} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{\underline{B}_{\underline{\nu}}})$$

Since $\alpha_{3/1}(\Pi_{\underline{x}}) = \Pi_{\underline{x}}$, by CombGC,

$\beta_{3/2}^{\underline{\nu}}$ stabilizes $\Pi_{\underline{B}_{\underline{\nu}}}$-conj classes of



$\Pi_{\underline{E}_{\underline{x}}}, \; \Pi_{\underline{L}_{\underline{x}}}, \; \Pi_{\underline{\nu}_{\underline{x}}}, \; \Pi_{\underline{\mu}_{\underline{x}}}$

Thus, we obtain $\beta_{3/2}^{\underline{L}} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{\underline{L}_{\underline{x}}})$

[cf. $N_{\Pi_{\underline{B}_{\underline{\nu}}}}(\Pi_{\underline{L}_{\underline{x}}}) = \Pi_{\underline{L}_{\underline{x}}}$]

Question $\beta_{3/2}^{\underline{L}} \overset{?}{=} \beta_{3/2}^{\underline{M}}\Big|_{\Pi_{\underline{L}_{\underline{x}}}}$

$\mathrm{Out}^{\mathrm{FC}}$

Note: we ma...

to "β_2^{tp}...

observe:

omb GC,

sses of

$\ldots, \Pi_{\underline{v}_{\underline{x}}}, \Pi_{\underline{\Delta}_{\underline{x}}}$

$\underline{x}$)

<u>observe</u>: Let $G_1, G_2 = \pi_1(\text{tripod})$

p_1, p_2 : geometric outer isom, δ : commutes with geom outer aut of G_2

$$\begin{tikzcd} G_1 \arrow[r, "\gamma_1 \in \mathrm{Out}(G_1)", "\sim"'] \arrow[d, "p_1"', "\wr"] \arrow[dr, phantom, "\circlearrowright"] & G_1 \arrow[d, "p_1"', "\wr"] \\ G_2 \arrow[r, "\sim", "\delta"'] & G_2 \end{tikzcd} \qquad \begin{tikzcd} G_1 \arrow[r, "\gamma_2 \in \mathrm{Out}(G_1)", "\sim"'] \arrow[d, "p_2"', "\wr"] \arrow[dr, phantom, "\circlearrowright"] & G_1 \arrow[d, "\wr", "p_2"'] \\ G_2 \arrow[r, "\sim", "\delta"'] & G_2 \end{tikzcd}$$

$\Rightarrow \gamma_1 = \gamma_2$

On the other hand,

$$\begin{tikzcd} \Pi_{\underline{L}_{\underline{x}}} \arrow[r, "\sim"] \arrow[d, "\wr"] & \Pi_{\underline{L}_{\underline{x}}} \arrow[d, "\wr"] & \beta^{\underline{L}}_{\gamma_2} \arrow[l, bend right] \\ \Pi_2^{tpd} \arrow[r, "\beta_2^{tpd}"] \arrow[d, "\sigma"', "\wr"] \arrow[dr, phantom, "\circlearrowright"] & \Pi_2^{tpd} \arrow[d, "\wr", "\sigma"'] & \\ \Pi_2^{tpd} \arrow[r, "\sim", "\beta_2^{tpd}"'] \arrow[d] & \Pi_2^{tpd} \arrow[d] & \\ \Pi_1^{tpd} \arrow[r, "\sim"] & \Pi_1^{tpd} & \beta_1^{tpd} \arrow[ll, bend left] \end{tikzcd} \qquad \begin{tikzcd} \Pi_{\underline{L}_{\underline{x}}} \arrow[r, "\beta^{\underline{M}}_{\gamma_2}|_{\Pi_{\underline{L}_{\underline{x}}}}", "\sim"'] \arrow[d, "\underline{P}_{23}"', "\wr"] \arrow[dr, phantom, "\circlearrowright"] & \Pi_{\underline{L}_{\underline{x}}} \arrow[d, "\wr", "\underline{P}_{23}"'] \\ \Pi_1^{tpd} \arrow[r, "\sim", "\beta_1^{tpd}"'] & \Pi_1^{tpd} \end{tikzcd}$$

, where $\sigma \in \mathrm{Out}(\Pi_2)$ is an outer modular sym s.t.

$$\begin{tikzcd} \Pi_{\underline{L}_x} \arrow[r, hook] \arrow[rrr, bend right] & \Pi_2^{\mathrm{tpd}} \arrow[r, "\sim"', "\sigma"] & \Pi_2^{\mathrm{tpd}} \arrow[r, two heads, "\mathrm{pr}_1"'] & \Pi_1^{\mathrm{tpd}} \end{tikzcd} : \text{"geom isom"}$$

//

Thus, by gluing $\beta_{3/2}^{\mu} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{\underline{B}_{\mu}})$

$\beta_{3/2}^{\nu} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{\underline{B}_{\nu}})$ along $\Pi_{\underline{L}_x}$,

we obtain $\underbrace{\beta_{3/2} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{3/2})}$

with ou

(claim follows

$\rightsquigarrow$ by claim, $\Pi_{2/1}$

$\downarrow \wr$

$\Pi_{2/}$

(Note : $\Pi_{2/1}$ is topo

$\rightsquigarrow$ we obtain $\beta_{3/1}$

ter modular sym s.t.

Π_1^{tpd} : "geom isom"

//

μ)

ν) along $\Pi_{\underline{L}_{\underline{x}}}$,

)

claim $\beta_{3/2} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{3/2})$ is compatible, relative to $\alpha_{2/1}$,

with outer actions $\Pi_{E_x} \longrightarrow \mathrm{Out}(\Pi_{3/2})$

$\Pi_{F_x} \longrightarrow \mathrm{Out}(\Pi_{3/2})$

(claim follows essentially from the construction of $\beta_{3/2}$ from $\beta^{\mu}_{3/2}$, $\beta^{\nu}_{3/2}$)

$\rightsquigarrow$ by claim,

$$\begin{array}{ccc} \Pi_{2/1} & \longrightarrow & \mathrm{Out}(\Pi_{3/2}) \\ \wr\downarrow \alpha_{2/1} & & \wr\downarrow \mathrm{Inn}(\beta_{3/2}) \\ \Pi_{2/1} & \longrightarrow & \mathrm{Out}(\Pi_{3/2}) \end{array}$$

commutes.

(Note : $\Pi_{2/1}$ is topologically generated by Π_{E_x}, Π_{F_x})

$\rightsquigarrow$ we obtain $\beta_{3/1} \in \mathrm{Out}^{\mathrm{FC}}(\Pi_{3/1})$

On the oth

$\Pi_{\underline{L}_x}$

Π_2^{tpd}

σ

Π_2^{tpd}

$\rightsquigarrow$ On t

mmutes.

$, \Pi_{F_x})$

$\rightsquigarrow$ On the other hand, since $\mathrm{Out}^{FC}(\Pi_{3/1}) \to \mathrm{Out}^{FC}(\Pi_{2/1})$ inj

$$\begin{array}{ccc} \Pi_1 & \longrightarrow & \mathrm{Out}(\Pi_{3/1}) \\ \wr\downarrow \alpha_1 & & \wr\downarrow \mathrm{Inn}(\beta_{3/1}) \\ \Pi_1 & \longrightarrow & \mathrm{Out}(\Pi_{3/1}) \end{array}$$ commutes.

$\rightsquigarrow$ we obtain $\beta_3 \in \mathrm{Out}^{FC}(\Pi_3)$ s.t. maps to $\beta_2 \in \mathrm{Out}^{FC}(\Pi_2)$ //